10.02 Present Value Tables: Time Value of Money

To determine the exact selling price of a bond requires the use of present value concepts. (ASC 835) Money that is received at a future date is less valuable than money received immediately, and present value concepts relate future cash flows to the equivalent present dollars. Present value is defined as the current measure of an estimated future cash inflow or outflow, discounted at an interest rate for the number of periods between today and the date of the estimated cash flow. Many decisions require adjustments related to the time value of money:

- **Present Value of Amount (lump sum)** – This is used to examine a single cash flow that will occur at a future date and determine its equivalent value today. The amount one needs to invest today, for how many years, at what interest rate, to get $1 back in the future.
- **Present Value of Ordinary Annuity** – This refers to repeated cash flows on a systematic basis, with amounts being paid at the *end* of each period (it may also be known as an **annuity in arrears**). Bond interest payments are commonly made at the end of each period and use these factors.
- **Present Value of Annuity Due (Now)** – This refers to repeated cash flows on a systematic basis, with amounts being paid at the *beginning* of each period (it may also be known as an **annuity in advance** or special annuity). Rent payments are commonly made at the beginning of each period and use these factors.
- **Future Values (compound interest)** – These look at cash flows and project them to some future date and include all three variations applicable to present values. This is the amount that would accumulate at a future point in time if $1 were invested now. The future value factor is equal to 1 divided by the present value factor. For example, an investment of $10,000 in two years at 10% would accumulate to the principal multiplied by the future value factor. In this case the $10,000 × 1/0.8265 = $12,100.

Present and Future Value Tables*

Future Value (Amount) of $1

(n) Periods	6%	8%	10%	12%	15%
1	1.060	1.080	1.100	1.120	1.150
2	1.124	1.166	1.210	1.254	1.323
3	1.191	1.260	1.331	1.405	1.521
4	1.262	1.360	1.464	1.574	1.749
5	1.338	1.469	1.611	1.762	2.011
10	1.791	2.159	2.594	3.106	4.046
15	2.397	3.172	4.177	5.474	8.137
20	3.207	4.661	6.728	9.646	16.367
30	5.743	10.063	17.449	29.960	66.212
40	10.286	21.725	45.259	93.051	267.864

Present Value of $1

(n) Periods	6%	8%	10%	12%	15%
1	0.943	0.926	0.909	0.893	0.870
2	0.890	0.857	0.826	0.797	0.756
3	0.840	0.794	0.751	0.712	0.658
4	0.792	0.735	0.683	0.636	0.572
5	0.747	0.681	0.621	0.567	0.497
10	0.558	0.463	0.386	0.322	0.247
15	0.417	0.315	0.239	0.183	0.123
20	0.312	0.215	0.149	0.104	0.061
30	0.174	0.099	0.057	0.334	0.015
40	0.097	0.046	0.022	0.011	0.004

Future Value (Amount) of an Ordinary Annuity of $1

(n) Periods	6%	8%	10%	12%	15%
1	1.000	1.000	1.000	1.000	1.000
2	2.060	2.080	2.100	2.120	2.150
3	3.184	3.246	3.310	3.374	3.473
4	4.375	4.506	4.641	4.779	4.993
5	5.637	5.867	6.105	6.353	6.742
10	13.180	14.486	15.937	17.549	20.304
15	23.276	27.152	31.772	37.280	47.580
20	36.786	45.762	57.275	72.052	102.444
30	79.058	113.283	164.494	241.333	434.745
40	154.762	259.056	442.592	767.091	1779.090

Present Value of an Ordinary Annuity of $1

(n) Periods	6%	8%	10%	12%	15%
1	0.943	0.926	0.909	0.893	0.870
2	1.833	1.783	1.736	1.690	1.626
3	2.673	2.577	2.487	2.402	2.283
4	3.465	3.312	3.170	3.037	2.855
5	4.212	3.993	3.791	3.605	3.352
10	7.360	6.710	6.144	5.650	5.019
15	9.712	8.559	7.606	6.811	5.847
20	11.470	9.818	8.514	7.469	6.259
30	13.765	11.258	9.427	8.055	6.566
40	15.046	11.924	9.779	8.243	6.642

Present Value of an Annuity Due of $1

(n) Periods	6%	8%	10%	12%	15%
1	1.000	1.000	1.000	1.000	1.000
2	1.943	1.926	1.909	1.893	1.870
3	2.833	2.783	2.736	2.690	2.626
4	3.673	3.577	3.487	3.402	3.855
5	4.465	4.312	4.170	4.037	3.855
10	7.802	7.247	6.759	6.328	5.772
15	10.295	9.244	8.367	7.628	6.724
20	12.158	10.604	9.365	8.366	7.198
30	14.591	12.158	10.370	9.022	7.550
40	15.949	12.879	10.757	9.233	7.638

** All values rounded to the nearest thousandth of a percent*

Converting From One Annuity to Another

In some cases, when attempting to determine the present value of a stream of interest payments, an ordinary annuity, the only table available may be for an annuity due. To determine the present value of the ordinary annuity, either:

- Use the factor for 1 more period and subtract 1.0 from it, or
- Use the factor for the appropriate number of periods and divide it by 1 + the interest rate
  - The factor for an ordinary annuity for 4 years at 8% is 3.312.
  - This can be derived by using the factor for 5 years for an annuity due, 4.312, and subtracting 1.0 to get 3.312.
  - This can also be derived by dividing the factor for an annuity due of 4 years, 3.577, by 1 + the interest rate, or 1.08, to get 3.312.

In other cases, when attempting to determine the present value of a stream of rent payments, an annuity due, the only table available may be for an ordinary annuity. To determine the present value of the annuity due, either:

- Use the factor for 1 less period and add 1.0 to it, or
- Use the factor for the appropriate number of periods and multiply it by 1 + the interest rate
  - The factor for an annuity due for 4 years at 8% is 3.577.
  - This can be derived by using the factor for 3 years for an ordinary annuity, 2.577, and adding 1.0 to get 3.577.
  - This can also be derived by multiplying the factor for an ordinary annuity of 4 years, 3.312, by 1 + the interest rate, or 1.08, to get 3.577.

Actual factors for $1 are typically provided in tables to be multiplied by the cash flows in exam problems.

Assume that a company can earn 10% on its money. If it had to wait one year to receive a dollar, that would be the equivalent to them of 91 cents today (rounding all information to the nearest penny). The reason is that 91 cents invested at 10% would earn approximately 9 cents over the next year and become a dollar. The way this relationship is expressed is by saying that the present value of 1 at 10% for 1 period = 0.91.

For multiple years at 10%, the factors (rounded) are:

Years	**Factor**
1	0.91
2	0.83
3	0.75
4	0.68
5	0.62
Ordinary Annuity	**3.79**

An ordinary annuity refers to payments being made at the end of each period and is simply the sum of the value of each of the payments. In the above, the present value of an ordinary annuity of 1 at 10% for 5 periods = 3.79, meaning that getting one dollar each year for the next 5 years is the equivalent of getting $3.79 immediately. Another way to express it is to say that a person who paid $3.79 today to obtain an annuity of $1 per year for the next 5 years is earning a 10% rate of return on their investment.

Assume the following facts on the issuance of a single bond:

Face Value	$1,000
Stated Rate	8%
Effective Rate	10%
Issue Date	1/1/X1
Pay Dates for Interest	12/31
Due Date for Principal	12/31/X5
PV of 1 at 10% for 5 periods	0.62
PV of ordinary annuity at 10% for 5 periods	3.79

To determine the selling price of this term bond on 1/1/X1, the interest payments of $1,000 × 8% = $80 per year and the principal payment of $1,000 due in 5 years will be discounted at the effective rate of return of 10%, as follows:

Item	**Amount**	**PV Factor**	**Present Value**
Principal	$1,000	0.62	$620
Interest (Annuity)	$80	3.79	$304
Total			$924

Notice that, as expected, the selling price of the bond is less than face value, because the effective rate of interest of 10% exceeds the stated rate of 8%.

The entry to record the issuance is as follows:

1/1/X1		
Cash	924	
Unamortized discount	76	
Bond payable		1,000

Occasionally, a company will issue a zero-coupon bond, which refers to a bond that pays no periodic interest (0% coupon rate of interest). The bondholder only receives the face value of the bond at maturity.